THE MOIRÉ METHOD IN THE STUDY OF DYNAMIC DEFORMATION PROCESSES IN ELASTIC-PLASTIC BODIES

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The moiré method is widely used in experiments studying the processes of mechanical metal working, and is described in detail in the literature. Quasistatic and nonstationary deformation of transparent bodies has been studied using the moiré method [1, 2]. This method was first used by us in [3] to analyze dynamic processes (specifically, spallation) in metal plates. The moiré method makes it possible to qualitatively and quantitatively estimate the stress—strain state of a plate during staged or single-pass loading. In combination with other experiments and numerical calcualtions, the method helps to explain not only the character of the deformation, but also the failure processes which occur during mechanical working of metals.

In this work, we present a system solving the equations for a description of the stress—strain state of the body, with the help of experimental information based on the moiré method, at large plastic strains during staged loading. We introduce an example of deformation and failure of an aluminum plate during its interaction with a rigid cylindrical open die.

1. For staged loading of an elastic-plastic body, we examine three basic configurations: initial (natural) with location radius-vector ${}^0\mathbf{r} = {}^0a^i\mathbf{e}_i$, current (stage n − 1), $\mathbf{r} = a^i\mathbf{e}_i$, and current (stage n) $\mathbf{R} = x^i\mathbf{e}_i$. Here $\mathbf{e}_i$ are unit orthogonal reference vectors, which are independent of the coordinates.

The unloading configuration is not studied, since it is impossible to distinguish between the elastic and plastic displacement components in the experiment, due to the great nonuniformity of the strains throughout the volume of the body. We will only assume that for any stage of the deformation, it is possible to neglect the elastic displacements and their gradients in comparison with their plastic counterparts.

Let us suppose that the moiré method makes it possible to determine the total increment in the displacements between loading stages at any point in a given region of the body. Sometimes this assumption is valid, for example, during axisymmetric deformation, or the application of a raster on a meridional section inside a body [3].

Thus, during application of stage n of a deformable raster initially, and on a deformed raster of stage n − 1, we have a displacement field ${}^0\mathbf{u}$ with respect to the initial state in coordinates x^i, and (incremental) displacement field $\mathbf{u}$ respect to the n − 1 configuration, also in coordinates x^i:

$$\mathbf{R}(x^i) = {}^0\mathbf{r}({}^0a^i) + {}^0\mathbf{u}(x^i),\ \mathbf{R}(x^i) = \mathbf{r}(a^i) + \mathbf{u}(x^i). \tag{1.1}$$

We introduce the local reference vectors

$$\mathbf{R}_i = \frac{\partial \mathbf{R}}{\partial x^i} = \mathbf{e}_i,\quad \mathbf{r}_i = \frac{\partial \mathbf{r}}{\partial x^i} = \mathbf{R}_i - \frac{\partial \mathbf{u}}{\partial x^i} = \mathbf{R}_j\left(\delta_{ij} - \frac{\partial u^j}{\partial x^i}\right). \tag{1.2}$$

The physical meaning of the second relation in (1.2) lies in the existance of an arbitrary set of reference vectors $\mathbf{r}_i$, frozen in the material, such that during motion, $\mathbf{u}(x^i)$ transforms to the orthonormal basis $\mathbf{R}_i$.

We introduce the Hamiltonian operator in the current configurations n − 1 and n [4]:

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$$\widehat{\nabla} = \mathbf{r}^i \frac{\partial}{\partial x^i}, \quad \nabla = \mathbf{R}^i \frac{\partial}{\partial x^i} \tag{1.3}$$

($\mathbf{r}^i$, $\mathbf{R}^i$ are the bases related to $\mathbf{r}_i$, $\mathbf{R}_i$).

Applying operator (1.3) to the vector location in configurations n − 1 and n, we obtain the tensor gradients

$$\widehat{\nabla}\mathbf{r} = \mathbf{r}^i\mathbf{r}_i = \widehat{\mathbf{E}},\ \widehat{\nabla}\mathbf{R} = \mathbf{r}^i\mathbf{R}_i,\ \nabla\mathbf{r} = \mathbf{R}^i\mathbf{r}_i,\ \nabla\mathbf{R} = \mathbf{R}^i\mathbf{R}_i = \mathbf{E},$$

where $\widehat{\mathbf{E}}$, $\mathbf{E}$ are the matrix tensors in configurations n − 1 and n. Specifically, $\mathbf{E} = \delta_{ij}\mathbf{e}_i\mathbf{e}_j$.

We introduce the Almanza measure of strain in the current configuration n with respect to that of the n − 1 and the natural configurations:

$$\mathbf{g} = \nabla\mathbf{r} \cdot \nabla\mathbf{r}^{\mathrm{T}},\ {}^0\mathbf{g} = \nabla^0\mathbf{r} \cdot \nabla^0\mathbf{r}^{\mathrm{T}}. \tag{1.4}$$

The stress state in the strained configuration n is characterized by the Cauchy stress tensor $\mathbf{T} = t^{sk}\mathbf{R}_s\mathbf{R}_k$, which, in the absence of body forces, satisfies the equilibrium equation

$$\nabla \cdot \mathbf{T} = \mathbf{0}. \tag{1.5}$$

The general expression for the connection between stresses and strains is usually written in the form [4] $\mathbf{T} = \mathbf{F}(\nabla\mathbf{R})$, However, in this case it is not possible to apply such a formula, since the Cauchy–Green strain is a unit tensor $\mathbf{G} = \widehat{\nabla}\mathbf{R} \cdot \widehat{\nabla}\mathbf{R}^{\mathrm{T}} = \delta_{ij}\mathbf{r}^i\mathbf{r}^j$. This follows from the choice of basis $\mathbf{R}_i$, in configuration n, which plays the role of a reference frame. Therefore, we will write the governing relations in the form

$$\mathbf{T} = \mathbf{F}(\nabla\mathbf{r}). \tag{1.6}$$

After a series of transformations, the requirement of material indifference of stress tensor $\mathbf{T}$ results in the following expression for (1.6):

$$\mathbf{T} = \mathbf{O} \cdot \mathbf{F}(\mathbf{V}) \cdot \mathbf{O}^{\mathrm{T}}. \tag{1.7}$$

Here, $\mathbf{O}$ is an orthogonal tensor, associated with the strain; $\mathbf{V}$ is a positive-definite symmetric tensor.

Note that the expression for polar expansion of the gradient of the position $\nabla\mathbf{r} = \mathbf{O} \cdot \mathbf{V}$ does not coincide with the notation of the traditional expression [4], where the position gradient $\widehat{\nabla}\mathbf{R}$ is expanded.

2. We introduce an expression for the stress tensor $\mathbf{T}$ of the n-th stage by means of the state in the preceding stage. We use the approach, expounded in [5], which includes the expansion of (1.7) in a series. Unlike [5], we account for terms of second order in the expansion. This makes it possible to use the expansion for the small but finite transformation from configuration n − 1 to n. Applying the operator ∇ on the second relation in (1.1), we obtain $\nabla\mathbf{r} = \mathbf{E} - \nabla\mathbf{u}$. Assuming the relative proximity of configurations n − 1 and n, we write the latter relation in the form $\nabla\mathbf{r} = \mathbf{E} - \delta\nabla\mathbf{u}$ (δ is a small parameter).

We expand all kinematic variables in (1.7) in a series in δ and keep terms of order $\sim\delta^2$. So, for the definition of $\mathbf{V}$, we use the relation

$$(\mathbf{V})^2 = \nabla\mathbf{r}^{\mathrm{T}} \cdot \nabla\mathbf{r} = \mathbf{E} - (\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})\delta + \nabla\mathbf{u}^{\mathrm{T}} \cdot \nabla\mathbf{u}\delta^2 \equiv \mathbf{f}(\delta),$$

$$\mathbf{V} \approx \mathbf{V}(\delta = 0) + \frac{\partial\mathbf{V}}{\partial\delta}(\delta = 0)\,\delta + \frac{1}{2}\frac{\partial^2\mathbf{V}}{\partial\delta^2}(\delta = 0)\,\delta^2,$$

which, after transformation, gives

$$\mathbf{V} = \mathbf{E} - \boldsymbol{\varepsilon}\delta + \frac{1}{2}(\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega} - \boldsymbol{\Omega}\cdot\boldsymbol{\varepsilon} - \boldsymbol{\Omega}^2)\,\delta^2 \tag{2.1}$$

$$\left(\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}}\right),\ \boldsymbol{\Omega} = \frac{1}{2}\left(\nabla\mathbf{u} - \nabla\mathbf{u}^{\mathrm{T}}\right)\right).$$

Similarly, we define $\mathbf{O}$ from the equation $\mathbf{O}\cdot\mathbf{V}=\mathbf{E}-\delta\nabla\mathbf{u}$:

$$\mathbf{O}=\mathbf{E}-\Omega\delta+\frac{1}{2}(\Omega^2-\boldsymbol{\varepsilon}\cdot\Omega-\Omega\cdot\boldsymbol{\varepsilon})\,\delta^2. \tag{2.2}$$

The expansion for $\mathbf{F}$ has the form

$$\mathbf{F}(\mathbf{V})\approx\mathbf{T}_{n-1}-\mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}\delta+\frac{1}{2}\mathbf{L}_2(\boldsymbol{\varepsilon})\,\delta^2, \tag{2.3}$$

where $\mathbf{T}_{n-1}$ is the Cauchy stress tensor in configuration n − 1; $\mathbf{L}_0$ is a tensor of rank four; $\mathbf{L}_2$ is a tensor-operator of second rank, which is nonlinear in $\boldsymbol{\varepsilon}$.

Substituting (2.1)-(2.3) into (1.7) and transforming, we obtain a relation between the stresses in configurations n and n − 1 (the symbol δ is omitted):

$$\begin{aligned}\mathbf{T}_n\approx\mathbf{T}_{n-1}-\boldsymbol{\Omega}\cdot\mathbf{T}_{n-1}+\mathbf{T}_{n-1}\cdot\boldsymbol{\Omega}-\mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}+\boldsymbol{\Omega}\cdot\mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}-\mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}+\\+\frac{1}{2}\mathbf{L}_2(\boldsymbol{\varepsilon})-\boldsymbol{\Omega}\cdot\mathbf{T}_{n-1}\cdot\boldsymbol{\Omega}+\frac{1}{2}(\boldsymbol{\Omega}^2-\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}-\boldsymbol{\Omega}\cdot\boldsymbol{\varepsilon})\cdot\mathbf{T}_{n-1}+\frac{1}{2}\mathbf{T}_{n-1}\cdot(\boldsymbol{\Omega}^2+\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}+\boldsymbol{\Omega}\cdot\boldsymbol{\varepsilon}).\end{aligned} \tag{2.4}$$

Here, terms with $\boldsymbol{\Omega}$ govern the influence of rotation on the change in the components of the "frozen-in" tensor $\mathbf{T}_{n-1}$; terms $\mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}$ and $(1/2)\mathbf{L}_2(\boldsymbol{\varepsilon})$ determine the contribution of pure strains; the remaining terms are mutual influence of pure strains and rotations between configurations n − 1 and n.

In (2.4), when accounting for only terms linear in δ and in the absence of pure deformations $\boldsymbol{\varepsilon}\equiv 0$, there is a difference in sign from the commonly used expression [4, 6]. This is explained by the choice of configurations and the system of basis vectors: in the moiré method, the displacement field $\mathbf{u}(x^i)$ is known in the coordinates x^i of configuration n, during the transformation from configuration n − 1 to n.

All quantities in (2.4) are relative to the basis $\mathbf{R}_i$. To transform to the next stage, there is a change in configuration. Therefore, for the application of (2.4), the tensor $\mathbf{T}_{n-1}$, which was determined in the basis $\mathbf{r}_i$ in the preceding step, must be transformed to the basis $\mathbf{R}_i$: $\mathbf{T}_{n-1}=t^{ij}_{n-1}\mathbf{r}_i\mathbf{r}_j={}^{R}t^{ij}_{n-1}\mathbf{R}_i\mathbf{R}_j$ (${}^{R}t^{ij}_{n-1}$ are components of the tensor $\mathbf{T}$ in configuration n − 1 with respect to the basis of configuration n). Using (1.2), we find

$${}^{R}t^{ks}_{n-1}=t^{ks}_{n-1}-t^{kj}_{n-1}\frac{\partial u^s}{\partial x^j}-t^{is}_{n-1}\frac{\partial u^k}{\partial x^i}+t^{ij}_{n-1}\frac{\partial u^k}{\partial x^i}\frac{\partial u^s}{\partial x^i}. \tag{2.5}$$

3. We distinguish between the spherical and deviatoric parts of these tensors:

$$\mathbf{T}=p\mathbf{E}+\mathbf{T}',\ \ \mathbf{L}_0\cdot\cdot\boldsymbol{\varepsilon}=l_0(\boldsymbol{\varepsilon})\,\mathbf{E}+\mathbf{L}_0'\cdot\cdot\boldsymbol{\varepsilon},\ \ \mathbf{L}_2(\boldsymbol{\varepsilon})=l_2(\boldsymbol{\varepsilon})\,\mathbf{E}+\mathbf{L}_2(\boldsymbol{\varepsilon}),$$

substitute this into (2.4), and transforming, obtain

$$p_n=p_{n-1}-l_0(\boldsymbol{\varepsilon})+\frac{1}{2}l_2(\boldsymbol{\varepsilon}); \tag{3.1}$$

$$\begin{aligned}\mathbf{T}_n'=\mathbf{T}_{n-1}'+\mathbf{T}_{n-1}'\cdot\boldsymbol{\Omega}-\boldsymbol{\Omega}\cdot\mathbf{T}_{n-1}'-\mathbf{L}_0'\cdot\cdot\boldsymbol{\varepsilon}+\boldsymbol{\Omega}\cdot\mathbf{L}_0'\cdot\cdot\boldsymbol{\varepsilon}-\mathbf{L}_0'\cdot\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}+\\+\frac{1}{2}\mathbf{L}_2'(\boldsymbol{\varepsilon})-\boldsymbol{\Omega}\cdot\mathbf{T}_{n-1}'\cdot\boldsymbol{\Omega}+\frac{1}{2}(\boldsymbol{\Omega}^2-\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}-\boldsymbol{\Omega}\cdot\boldsymbol{\varepsilon})\cdot\mathbf{T}_{n-1}'+\\+\frac{1}{2}\mathbf{T}_{n-1}'\cdot(\boldsymbol{\Omega}^2+\boldsymbol{\varepsilon}\cdot\boldsymbol{\Omega}+\boldsymbol{\Omega}\cdot\boldsymbol{\varepsilon}).\end{aligned} \tag{3.2}$$

Relation (3.1) shows that rotation does not influence the mean pressure p. Therefore, in the case of a stressed state which is close to hydrodynamic, accounting for the rotation of material particles in the medium does not lead to a significant correction.

The terms ℓ_2 and $\mathbf{L}_2'$ in (3.1), (3.2) reflect physical nonlinearity. However, over a large range of pressures, $\ell_2 \ll \ell_0$ [7]. Taking the term $\mathbf{L}'_2$ into account is necessary for materials which exhibit inelastic properties. We introduce the notation $\mathbf{L}'\equiv-\mathbf{L}_0'\cdot\cdot\boldsymbol{\varepsilon}+\frac{1}{2}\times\mathbf{L}_2'(\boldsymbol{\varepsilon})$.

Following [8], we write for the variation in flow theory

$$de^p_{ij}=d\lambda S_{ij},\ \ dA_p=S_{ij}de^p_{ij}=2d\lambda\sigma_u^2,$$

where $\mathbf{e} = \mathrm{dev}(\boldsymbol{\varepsilon})$; dA_p is the incremental work of plastic deformation; σ_u is the intensity of the tangential stresses; S_{ij} are components of the tensor T'. Since $dA_p = \sigma_u d^0e_u^p$, then

$$S_{ij} = \frac{de_{ij}^p}{d\lambda} = 2de_{ij}^p \frac{\sigma_u\left({}^0e_u^p\right)}{de_u^p}. \tag{3.3}$$

Increments de_{ij}^p and de_u^p have the sense of deformations $\mathbf{e}^p = \mathrm{dev}\ (\boldsymbol{\varepsilon}^p)$ and correspondingly $e_u{}^p(\boldsymbol{\varepsilon}^p)$, obtained in terms of the displacement field $\mathbf{u}$. Then, (3.3) transforms into

$$\mathbf{L}' = 2\mathbf{e}\,\frac{\sigma_u\left({}^0e_u\right)}{e_u}, \tag{3.4}$$

where the index p has been omitted.

To compute ${}^0e_u$ in (3.4), it is necessary to use the measure of strain ${}^0\mathbf{g}$, which is determined by the gradient of the displacement ${}^0\mathbf{u}(x^i)$. Such an approach increases the amount of processed information by a factor of two. Another method of obtaining a measure of the total strain is to sum the strain tensors by stages with the appropriate transformation to the basis of the current configuration, in analogy to (2.5). This, however, increases the error which accumulates with each stage.

Relations (3.1) and (3.2) can be used in numerical algorithms to determine the stresses. When the initial information is determined with the help of the moiré method, the use of (3.1) is unjustified, for two reasons. In the first place, the error in the volume strains exceeds the value of the initial data. Secondly, after unloading, the volume strain is partially lost (complete unloading does not give residual stresses, which arise due to strong nonuniformities in plastic deformation throughout the volume of the body). Therefore, we use the method of seeking the mean pressure p with the help of the equilibrium equation (1.5), a method commonly used in mechanical metal working. Using $\mathbf{T} = p\mathbf{E} + \mathbf{T}'$, we transform (1.5):

$$\nabla \cdot (p\mathbf{E}) = -\nabla \cdot \mathbf{T}'. \tag{3.5}$$

Having previously determined the stress deviator tensor $\mathbf{T}'$ in configuration n according to (3.2), (3.4), we reduce the problem of determining p to integration of one of the differential equations in (3.5). The use of (3.5) introduces additional error in the determination of p, and thus in determining the normal components of the stress tensor in comparison to the strain and the stress deviator tensors. The additional error is introduced through the operation $\nabla \cdot \mathbf{T}'$, which assumes yet another numerical differentiation.

4. Processing of the experimental information is done in the "moiré" complex of programs, created specifically for this purpose. The complex includes modules for the reconstruction of displacements A, calclation of strains B, calculation of stresses C, and a graphics module D. All of the programs are formulated as independent subprograms, and can be used individually. The programs were created for the BÉSM-6 computer, and have been adapted to run on ES machines.

The input to module A consists of the coordinates (r_k, z_k) of the points of the moiré bands, the values of the displacements U_k in the bands, and the coordinates of the boundary of the region G. We have as output from the module the mesh functions U^{ij}, defined at the nodes of the rectangular mesh (i, j) inside G. Reconstruction of the values of U^{ij} is done with the help of two-dimensional splines of degree one [9], by the choice of optimal triangles containing the point (i, j). The numerical differentiation in module B is based on rational splines [9]. When the experimental information is limited and errors are introduced, the procedure of linear filtering is used to suppress nonphysical oscillations in the function U. Module A uses about 80% of the total time required for information processing.

The input of experimental information is accomplished by various means, mentioned in [3], including a semi-automated input regime into the computer.

5. As an example, we examine the results of analysis of moiré patterns obtained after impact of a rigid cylindrical open die with plane ends on an aluminum plate. The method used in the experiment is similar to that in [3]. Figure 1 shows the moiré pattern representing lines of equal axial (a) and radial (b) displacements. Directly under the die is a zone which is insensitive to moiré effect.

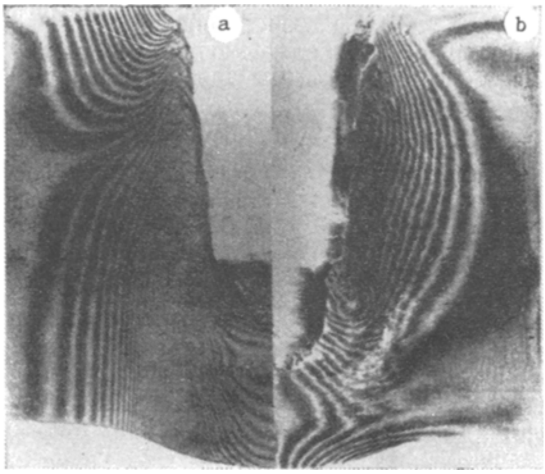

Fig. 1

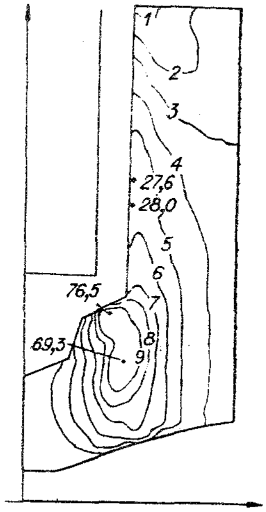

Fig. 2

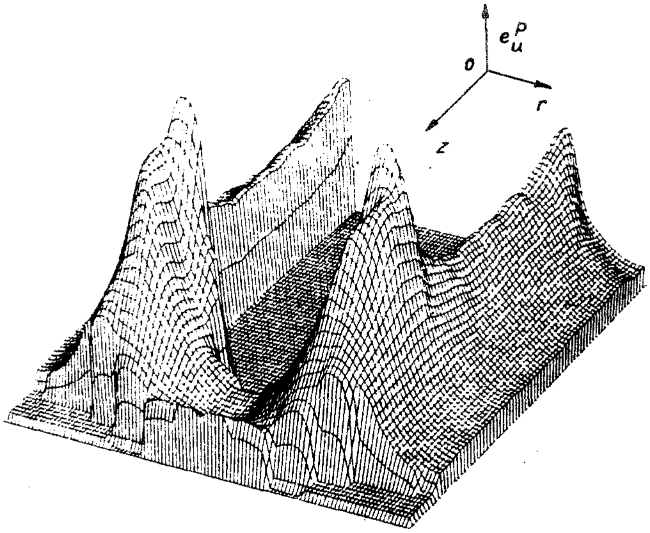

Fig. 3

Processing of the experimental information by the "moiré" complex makes it possible to determine all components of the tensors **A** and **T**. In all of the drawings, a contour line shows the position of the striker and delimits the boundary of the processed region. The back surface is located in the neighborhood of the origin of the coordinates and the vertical axis.

From the analysis, it is clear that the axial compression strain A_{zz} is distributed under the die, and along the side walls of the crater $A_{zz} > 0$. The maximum elongation, equal to 17%, is attained near the front surface of the plate in the "petalling" region, which is caused by the inertial ejection of material. The radial strain A_{rr} is negative almost everywhere, with a minimum near the side walls of the crater. A modest elongation (~5%) is observed at the rear bulge of the plate under the die.

Figure 2 shows contours of shear strain A_{rz}: the numbers 1-9 corresponds to strain levels frlom −20 to +60%, in steps of 10%. The maximum value (~76%) is attained near the crater angles, where shear cracks are then formed. The large shears are localized in a region not more than 5 mm wide, through which shears on the order of 30% emerge at the rear surface.

The surface of intensity of the shear strains is illustrated in Fig. 3 (the zone of moiré insensitivity is depicted by the zero level (the flat area)). The maximum value (90%) is reached near the crater angles, as in the case of the components A_{rz}. The strain intensity is localized in the neighborhood of the cylindrical surface, along which the plug forms. Comparison of Figs. 2 and 3 show that the shear strain makes the primary contribution to the strain intensity.

We can draw the following conclusions from an analysis of the stresses. All normal stresses directly under the crater and near its lateral surfaces are negative, reaching ~0.8

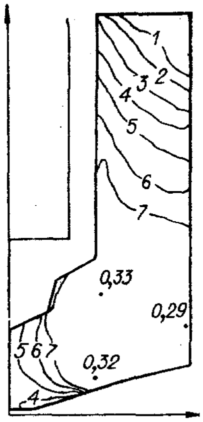

Fig. 4

GPa (under the crater) and 1.3 GPa (near the lateral surface). At the back surface, the circumferential and radial stresses are positive (~0.2-0.3 GPa), which can lead to the formation of cracks at either the bulge tip, or at a distance of 1.5-2 die radii from the axis of symmetry. Only shear cracks can form inside the plate. This system of cracks is located in the neighborhood of the maximum tangential stress.

Figure 4 shows contours of the tangential stress field. The numbers 2-6 correspond to levels frlom −0.2 to +0.2 GPa in steps of 0.1 GPa. Contour number 1 corresponds to 0.25 GPa, number 7 to 0.25 GPa. The final form of the crack differs from a straight line, due to the application of subsequent deformation in the course of penetration and rotation of material particles. The brittle failure cracks inside the plate can be formed, evidently, in the initial stages of penetration, as a result of the interaction of the stress waves.

In conclusion, we note that representation of the data on the character of plate deformation of the crater during penetration is obtained for the first time with the help of the moiré method. The method devised here can be used to study various quasistatic processes with staged loading, for example, in mechanical metal working, the study of technological operations, and so on.

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